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Using Corporate Inflation Protected Securities to Hedge Interest Rate Risk

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In the first half of 2008, commodity prices surged, the U.S. dollar weakened, and the International Monetary Fund announced that the inflation rate in developed economies was likely to reach its highest level since 1995.[1] But by the fall of 2008, the situation was dramatically different; the price of oil declined from $150 a barrel to less than $40 and the price of gasoline at the pump fell by more than 50% during the worst recession since the Great Depression. The attention of the Federal Reserve promptly turned from fighting inflation to preventing deflation, leading to a dramatic increase in the monetary base and the target Fed funds rate falling to below 25 basis points.

Such shifts in the outlook for inflation represent a significant risk for some companies, particularly those whose revenues and profits are negatively affected by increases in inflation and rates. For such companies, the use of long-term fixed-rate debt will provide at least a partial hedge against increased rates.

But less widely appreciated is that even companies whose profits move up and down with inflation face considerable risk from fluctuations in interest rates. The conventional wisdom holds that such companies effectively hedge their interest rate risk and stabilize their (after-interest) operating cash flow by using mainly floating-rate debt debt. This way, when inflation increases, the company's revenue stream and general asset values increase along with the interest rate and the dollars required to service the debt, and net income remains relatively stable.

But, as we argue in this article, even in these circumstances, the use of floating-rate debt still leaves the company exposed to increases in *real* interest rates. That is, if interest rates go up by more than the expected rate of inflation, a company with floating- rate debt will see its interest payments increase faster than its revenue, and the company's profitability will suffer.

This paper shows how inflation-sensitive companies such as utilities can use corporate inflation-protected securities (CIPS) to hedge their real interest rate risk as well as inflation risk. In addition to its hedging benefits, CIPS also have the potential to reduce borrowing costs by satisfying investor demand for inflation protection.[2] For example, during the inflationary cycle of 2008, Fannie Mae issued two inflation-linked notes, citing strong investor demand for securities offering a combination of inflation protection and high credit quality.[3] Similar events in 2003 and 2004 led several financial firms in the U.S., including Fannie Mae, Household Finance Corp., Sallie Mae, Morgan Stanley and Merrill Lynch, to issue CIPS.

The Market for Inflation Protection

In 1997, the federal government began issuing inflation-indexed bonds called Treasury Inflation Protected Securities, or TIPS. The face value of a TIPS bond is adjusted over time in response to movements in the economy's overall price level. A constant rate of inflation of 3%, for example, will result in the face value being increased each year by 3%. And, as a result, the bond's coupon payments will also go up by 3%, since they are the product of the coupon rate and the face value. Thus, the yield-to-maturity on TIPS is a guaranteed "real" rate of interest, and their holders are completely protected against unanticipated inflation.

Corporate inflation-protected securities, or CIPS, are similar to TIPS in that they offer investors a real rate of return that is protected from changes in inflation. But their structure differs from that of TIPS in a couple of ways detailed later in the paper.

During the past decade, demand for inflation-protected securities of any kind has grown largely as a result of two important developments. First, pension funds are increasingly seeking to hedge inflation risk as a result of changing regulatory requirements.[4] At the same time, both pensions and insurance companies have recognized the stabilizing benefits of inflation hedging, since their liabilities require a focus on real returns over long horizons. The low correlation of

1. *Wall Street Journal*, April 10, 2008, "Inflation, spanning globe, is set to reach decade high."

2. A potential source of lower funding costs is the inflation risk premium that investors are willing to pay in order to protect real returns. By embedding inflation insurance in their bonds, issuers can realize this premium, which has been estimated to be 50 to 100 basis points in the U.S. during 1953–1994. J.Y. Campbell and R.J. Shiller, "A Scorecard for Indexed Government Debt," *NBER Macroeconomics Annual*, Vol. 11 (1996), pp. 155–196.

3. Fannie Mae, "Update on Fannie Mae's Issuance of Structured Notes—With a Focus on CPI-Linked Notes," *Funding Notes for Fannie Mae's Investors and Dealers*, http://www.fanniemae.com/markets/debt/pdf/fundingnotes_07_08.pdf, July/August 2008

4. This is an important element of the market in the U.K., for example, where pension obligations have been tied to the Retail Price Index or Limited Price index. Public pension funds generally are often required to compensate retirees for inflation.

inflation-linked investments with equities and nominal bonds provides further risk reduction benefits for these investors. The second development is the linkage of savings accounts to inflation. For example, the return of the inflation linkage of French savings accounts has by itself added some $175 billion in inflation-linked liabilities that require inflation hedges.[5]

While sovereign markets have grown to meet this demand—reaching over $760 billion in 2005—the supply of bonds in the non-financial corporate market has lagged far behind.[6] To date, the largest non-financial corporate issuers have been U.K. utilities, companies whose billings are explicitly tied to inflation and whose CIPS issuance has accounted for over 80% of the global corporate market.[7]

In addition to (or instead of) meeting investor demand by issuing inflation-protected bonds, corporations can pay inflation in the swap market that has developed around institutions' desire to hedge inflation risk.[8] Banks and insurance companies with structured inflation payments that are not matched by traditional bonds have hedged these liabilities by investing in structured inflation swaps that match their liability cash flows.[9] The institutions receive an inflation-linked payment in exchange for a fixed rate payment. Corporations can convert their traditional fixed rate debt to inflation-linked debt by entering the other side of these transactions. An ongoing challenge in this market, however, has been the lack of inflation payers, resulting in higher prices for synthetic bonds than those observed in the cash market.[10] Corporate issuers could potentially exploit this market imbalance to lower their funding cost while simultaneously reducing risk.

The Impact of Real and Nominal Rate Changes on Firm Value

The sensitivity of a company's value or an investment project's NPV to changes in the real interest rate has been extensively analyzed.[11] Holding future cash flows constant, an increase in the required rate of return will cause the value to fall, with the magnitude of the change depending on the "duration" of the anticipated cash flows.[12] The present value of project cash inflows with a long duration will fall by a relatively large amount if the real interest rate rises, whereas a project whose inflows have a shorter duration will be less impacted. The adverse effect of a real interest rate increase on the NPV can be limited by issuing debt that reduces the duration gap close to zero.

An increase in nominal interest rates can result from an increase in either the real rate of interest or the expected rate of inflation, or both. A careful analysis of the impact of an increase in interest rates needs to identify the cause of the rate hike. A simple example illustrates the issues involved, contrasting the effects of an increase in the real interest rate with an increase in the inflation premium.

Consider a company with an asset that generates a single cash inflow of $1,000,000 in the future, with a present value of A and duration D_A. The company also has a single cash outflow liability of $500,000, with present value L and duration D_L. The equity value is the difference between A and L. The impact on the equity value of a change in the interest rate will depend on the relationship between the magnitude and durations of the future inflow and outflow.

As discussed extensively in the banking literature, a company's equity is protected against interest rate changes when its liabilities have maturities that set the "duration gap" equal to zero, where the duration gap is defined as $[D_A - D_L(L/A)]$.[13] Suppose our hypothetical company has chosen to issue debt with this property. Assume now that the interest rate used to discount all future cash flows is 4%, the asset's duration is 0.96 years, the liability's duration is 2 years, and initially there is no inflation. Using the 4% interest rate, the present value of the cash inflow is $A = 1,000,000/(1.04)^{0.96} = 963,048$ and the present value of the outflow is $L = 500,000/(1.04)^2 = 462,278$. Consequently, the equity (E) value is:

$$E = A - L = 963,048 - 462,278 = 500,770.$$

The firm is "immunized" against interest rate changes since the duration gap is equal to zero,

$$DGAP = D_A - (L/A)D_L = 0.96 - (462,278/963,048)2 = 0.$$

What happens to the company's value if the interest rate increases to 7.12%? If the rate climb is caused by a rise in the real interest rate, and if future cash flows are unaffected, then the company's equity value will not materially change as a result of the rate increase. Holding the future cash inflow and outflow constant, the new equity value equals:

$$E = 1,000,000/(1.0712)^{0.96} - 500,000/(1.0712)^2 = 500,363.$$

5. V. Armann, B. Benaben and B. Lambert, "Inflation Flows and Investment Strategies," in B. Benaben, Ed., *Inflation-Linked Products* (London: Risk Books, 2005).

6. R.T. Dalio and D.S. Bernstein, "The Benefits of Global Inflation-Indexed Bonds," in B. Benaben, Ed., *Inflation-Linked Products* (London: Risk Books, 2005).

7. Armann (2005), cited earlier. As noted previously, in the U.S., the largest issuers have been financial institutions.

8. The inflation swap market originated in the United Kingdom in the mid-1990s and began to grow rapidly in 1998 following France's first inflation-linked bond issuance.

9. Armann (2005), cited earlier.

10. Armann (2005), cited earlier.

11. L.D. Barney and M. Danielson, "Ranking Mutually Exclusive Projects: The Role of Duration," *The Engineering Economist*, Vol. 49, No. 1 (2004), pp. 43–61; L.D. Barney and H. White, "Project-Specific Financing and Interest Rate Risk in Capital Budgeting," *The Engineering Economist,* Vol. 48, No. 2 (2003), pp. 169–182; E. Blocher, and C. Stickney, "Duration and Risk Assessments in Capital Budgeting," *The Accounting Review*, Vol. 14, No. 1 (January 1979), pp. 180–188. D. Durand, "Payout Period, Time Spread and Duration: Aids to Judgment in Capital Budgeting," *Journal of Bank Research*, Vol. 4 (Spring 1974), pp. 20–34.

12. F.R. Macaulay, *Some Theoretical Problems Suggested by the Movements of Interest Rates, Bond Yields, and Stock Prices in the United States since 1856* (New York: Columbia University Press, 1938).

13. See G.O. Bierwag and G.C. Kaufman, "Duration Gap for Financial Institutions," *Financial Analysts Journal*, Vol. 41, No. 2 (March/April 1985), pp. 68–71.

But if the rise in the interest rate is caused by a higher inflation premium, the value impact is significantly greater. Using Fisher's equation, a real rate of interest of 4% combined with an inflation premium of 3% produces a nominal interest rate of 7.12%.[14] For many industrial companies, higher inflation that leads to rising interest rates will also be associated with higher future operating income. Starting with a real cash inflow of $1,000,000 and 3% inflation, the company's cash inflow is reasonably expected to grow to $1{,}000{,}000(1.03)^{0.96} = 1{,}028{,}783$. Even though the duration gap is zero, the value of equity is not immunized against changes in nominal interest rates, as the new equity after the jump in inflation is:

$$E = 1{,}028{,}783/(1.0712)^{0.96} - 500{,}000/(1.0712)^{2} = 527{,}307 .$$

The rise in the inflation premium benefits equity, as future cash inflows go up, while the debt payments remain fixed. Had inflation fallen, the impact on equity would be negative, as happens in deflationary environments when the prices of output fall. The point is that setting the duration gap equal to zero has not immunized the company's equity against changes in nominal interest rates caused by changes in inflation.

Traditional floating-rate debt will not solve the problem of immunization either. If the company issues floating-rate debt, then a rise in the real interest rate will result in higher cash outflows to service debt, but the company's operating income will not show an associated increase. Thus, the equity value will fall because of the combined impact of a higher interest rate applied to discount future cash inflows and higher cash outflows required to service the floating rate debt. The solution to immunizing against changes in both the real interest rate and the inflation premium is to use a combination of duration gap management and inflation-indexed debt, which we turn to next.

A Model of Interest Rate Risk in Capital Budgeting

In this section, we present a general model to show how changes in real interest rates contrast with changes in inflation premiums in a capital budgeting framework. The equity residual approach is used to calculate the net present value of a project.[15] The most commonly used technique to calculate net present values is the operating-cash flow approach, which completely ignores financing in the calculation of a project's cash flows. The less familiar equity-residual approach, however, explicitly accounts for payments to a company's debt-holders as part of a project's cash flow calculations. Since our main purpose is to describe how a company's investments can be financed so as to minimize inflation and interest rate risk, the equity-residual approach is preferred

The project has an initial cash outflow, $CF_0 < 0$, followed by subsequent operating cash inflows CF_t in each year t of the project's life. A portion of the project's cost is financed through borrowing, equal to B. Interest and principal payments on the debt in each year are represented by I_t and P_t, respectively, so the total debt payment in year t is $I_t + P_t$. For simplicity, we assume no corporate tax.

The period t denominator in the NPV calculation is $(1 + k_e)$, where k_e is the nominal cost of equity. Fisher's equation is used to decompose this into a real rate, r_e , and an inflation premium, π:

$$(1 + k_e)^t = (1 + r_e)^t (1 + \pi)^t .$$

With the equity residual approach, the project's NPV is given by Eq. (1):

$$NPV = CF_0 + B + \sum_{t=1}^{T} \frac{CF_t}{(1 + r_e)^t (1 + \pi)^t} - \sum_{t=1}^{T} \frac{(I_t + P_t)}{(1 + r_e)^t (1 + \pi)^t} \qquad (1)$$

Assuming inflation impacts all revenue and expenditures uniformly, the nominal operating cash inflow in year t can be thought of as the real cash flow measured in today's dollars, represented by CF_t^*, inflated for the appropriate number of years:

$$CF_t = CF_t^* (1 + \pi)^t .$$

Substituting this into Eq. (1) and cancelling the $(1 + \pi)^t$ terms gives

$$NPV = CF_0 + B + \sum_{t=1}^{T} \frac{CF_t^*}{(1 + r_e)^t} - \sum_{t=1}^{T} \frac{(I_t + P_t)}{(1 + r_e)^t (1 + \pi)^t} . \qquad (2)$$

To see how an increase in inflationary expectations impacts the net present value, Eq. (2) is differentiated with respect to π. As shown in Appendix A, the effect of a rise in the inflation premium depends on the present value of the loan payments (L) and the Macaulay duration of the loan payments (D_L):

$$\Delta NPV \approx \frac{D_L L}{(1 + \pi)} (\Delta \pi) \cdot \qquad (3)$$

Equation (3) suggests that a rise in nominal interest rates will increase the NPV of a capital budgeting project if the rate hike is caused by higher inflationary expectations. The

14. Using Fisher's equation, the nominal interest rate of 7.12 percent is calculated as (1.04)(1.03)-1.

15. The equity residual approach is explained in D. R. Chambers, R. S Harris and J.J. Pringle, "Treatment of Financing Mix in Analyzing Investment Opportunities," *Financial Management*, Vol. 11, No. 2 (Summer 1982), pp. 24–41.

magnitude of the increase hinges on how much of the project is financed by debt, as well as the debt's duration. All else equal, the longer the duration of the debt financing, the more shareholders stand to benefit from an increase in the inflation premium built into interest rates. Of course, the reverse is also true: The NPV will fall if unanticipated deflation emerges and the project was financed with long-term debt.

The impact of an increase in the real interest rate is found by differentiating Equation (1) with respect to r_e. It will depend on the duration and value of the debt, as well as the present value of the operating cash inflows (A) and the duration of the operating cash inflows (D_A). As shown in Appendix B,

$$\frac{\Delta NPV}{\Delta r_e} \approx -[D_A - D_L(L/A)]A\frac{1}{(1+r_e)}, \qquad (4)$$

where the duration gap is $[D_A - D_L(L/A)]$.[16] Using this result, previous research has shown how a project's financing can be structured to yield a zero duration gap, and thereby "immunize" the net present value against changes in the required rate of return.[17] But setting the duration gap equal to zero does not necessarily immunize the project's NPV from fluctuating interest rates due to a changing inflation premium. Immunizing against a changing inflation premium requires a company to borrow by issuing an inflation-protected debt security.

Immunizing Against Changes in the Inflation Premium

TIPS are bonds whose face value increases over time with inflation, and thus the bonds' coupon payments also go up. Letting FV^* represent the initial face value of a TIPS bond, the face value in a subsequent period t will be $FV_t = FV^*(1+\pi)^t$. Likewise, the bond's coupon payment will change over time, so by year t will equal $PMT_t = xFV_t = xFV^*(1+\pi)^t$. Appendix C demonstrates that by issuing bonds with such characteristics, inflation is eliminated from the NPV expression in Equation (1). Moreover, by issuing debt with the appropriate duration immunizes the project's NPV against changes in the real interest rate.

An Example of Immunization Using CIPS

CIPS differ from TIPS in that the entire inflation adjustment occurs in the coupon payment; the face value of the security does not change over time. The inflation rate for the prior 12 months is simply added to the original coupon rate. Thus, with CIPS the coupon payment in period t equals:

$$PMT_t = FV(x + \pi).$$

This difference between TIPS and CIPS has a significant impact on the securities' periodic cash flows, since the additive inflation term results in a much larger adjustment to the coupon than the multiplicative adjustment for TIPS. For example, if the initial interest rate is 2% and the inflation rate during the "look-back" period is 3%, the CIPS coupon payment will be 5% times the bond's face value, while the TIPS coupon payment is only 2.06% of the bond's original face (2.0% * 1.03).

With TIPS, inflation is compounded in the coupon over time. For instance, if inflation is again 3% in the second period, the TIPS coupon is 2.122% of the bond's original face amount (2.0% * 1.03^2), while the CIPS coupon remains constant at 5%. The result is that the CIPS coupon is more responsive to changes in inflation and far more volatile when the inflation rate is fluctuating, while the inflation protection in TIPS is concentrated in the principal adjustment.

Compared to TIPS, the CIPS coupon structure is attractive to taxable investors because the tax liability arising from a higher inflation rate more closely matches the increased cash flows associated with receipt of the higher coupon payments. Each year TIPS investors are taxed on the principal adjustment, but the bulk of the increased cash flow from the adjustment is deferred until the principal is collected at maturity. To the extent that investors are focused on the tax treatment, a corporate issuer can possibly lower its cost of debt by using bonds with the CIPS structure.

Another structural difference between the two securities concerns the frequency of coupon payments and lag of the inflationary adjustment. In a typical CIPS structure coupons are paid monthly and the inflation adjustment is based on the Consumer Price Index for All Urban Consumers (CPI-U) during the 12-month period ending three months prior to the coupon payment.[18] For example, consider a CIPS security in which the first monthly coupon payment is due in March. This payment will be a function of the inflation rate that prevailed during January through December of the preceding year.

TIPS pay semi-annual coupons, with the inflation adjustment based on the period ending one month prior to the coupon. The TIPS structure better matches the actual inflation experienced during the security's life, especially for short-term securities in an unstable inflation environment. In the example above, the March CIPS coupon is adjusted for inflation that occurred prior to the security's issuance. The investor is not compensated for inflation occurring solely during their investment horizon until the 15th coupon. If inflation has risen since the prior year, this will lower the real yield earned by the investor, since the coupons will be adjusted for the previously lower inflation level.

16. The right-hand side of Eq. (4) parallels the duration gap derived within the framework of bank interest rate risk management in G. O. Bierwag and G. C. Kaufman, "Duration Gap for Financial Institutions," *Financial Analysts Journal*, Vol. 41, No. 2 (March/April 1985), pp. 68–71. The result is obtained in the context of a capital budgeting problem in Barney and White (2003).

17. Barney and White (2003), cited earlier.

18. Inflation adjustments to both CIPS and TIPS are based on the CPI-U, which is issued monthly by the Bureau of Labor Statistics.

The level of the CIPS coupon at issuance will include, as compensation, the difference between historical and expected inflation, and trading prices after issuance reflect this same disparity. Similarly, CIPS investors benefit from the longer lookback period when inflation is decelerating. But the longer lookback is detrimental to corporate issuers seeking an inflation hedge, since a portion of the inflationary adjustment will not coincide with the period over which a project's cash flows are being affected by inflation.

To see how CIPS can immunize a company against changes in interest rates, consider a project whose operating cash flows depend on inflation as follows:

$$CF_0 = -100{,}000.00$$
$$CF_1 = +50{,}000.00(1+\pi)^1$$
$$CF_2 = +30{,}000.00(1+\pi)^2$$
$$CF_3 = +44{,}172.80(1+\pi)^3$$

For simplicity, we ignore some real-world complications with CIPS by assuming an annual coupon payment whose size depends upon the inflation rate occurring in the 12-month period immediately prior to the payment (that is, there is no lag). The CIPS is assumed to have a coupon rate of 10% when inflation is zero, a $60,000 face value, four years to maturity, and a selling price equal to the face value. The actual coupon payment in any year depends on the inflation rate π, and is given by $(0.10 + \pi)\,60{,}000$. The project's nominal cash flows using the equity residual approach are as follows:

$$CF_0^e = -40{,}000$$
$$CF_1^e = 50{,}000(1+\pi) - (0.10+\pi)60{,}000$$
$$CF_2^e = 30{,}000(1+\pi)^2 - (0.10+\pi)60{,}000$$
$$CF_3^e = 44{,}172.80(1+\pi)^3 - (0.10+\pi)60{,}000$$
$$CF_4^e = -(0.10+\pi)60{,}000 - 60{,}000$$

Since there is no operating cash inflow in year 4, the equity-residual cash outflow is simply the final coupon interest and principal payment on the bonds.

By using CIPS, the company has effectively immunized against changes in the real interest rate, as well as changes in interest rates due to inflation. To illustrate, we begin with an initial expectation of zero inflation and a real cost of equity of 15%. Then the combination of a zero duration gap and CIPS financing essentially immunizes the net present value from changes in the inflation rate or the real cost of equity.

To verify that the project financing has a zero duration gap, note that discounting the debt's cash flows at the real cost of equity gives

$$L = \sum_{t=1}^{4} \frac{6000}{(1.15)^t} + \frac{60{,}000}{(1.15)^4} = 51{,}435.06$$

and a duration of

$$D_L = \frac{\frac{1(6000)}{(1.15)^1} + \frac{2(6000)}{(1.15)^2} + \frac{3(6000)}{(1.15)^3} + \frac{4(66{,}000)}{(1.15)^4}}{51{,}435.06} = 3.44\ .$$

The present value of the operating cash flows A and the duration of the operating cash flows D_A are

$$A = \frac{50{,}000}{(1.15)^1} + \frac{30{,}000}{(1.15)^2} + \frac{44{,}172.80}{(1.15)^3} = 95{,}206.90\ ,$$

$$D_A = \frac{\frac{1(50{,}000)}{(1.15)^1} + \frac{2(30{,}000)}{(1.15)^2} + \frac{3(44{,}172.80)}{(1.15)^3}}{95{,}206.90} = 1.848\ .$$

Using these quantities, the project is immunized against changes in real interest rates, as the duration gap is nearly zero:

$$\left[D_A - (L/A)D_L\right] = 1.848 - \left(\frac{51{,}435.06}{95{,}206.90}\right)3.44 \approx 0\,.$$

Starting with zero inflation, the project's NPV for different values of the real cost of equity is shown below:

Real Cost of Equity	Net Present Value
13%	3,710.21
14%	3,751.97
15%	3,771.84
16%	3,771.42
17%	3,752.23

Changes in the expected rate of inflation will also have a minimal impact on the project's NPV.[19] To illustrate, suppose immediately after accepting the project the inflation rate goes up to 1% (from zero initially). The real cost of equity remains at 15%, and the nominal cost of equity will now be 16.15%. The project's equity-residual cash flows are now:

19. With TIPS the inflation premium entirely cancels out of the NPV equation, as shown in Appendix C. With CIPS the cancellation is not as neat, but the result is nearly the same: Inflation risk is essentially eliminated from the problem when issuing either type of security.

$CF_0^e = -40,000$

$CF_1^e = +43,900$

$CF_2^e = +24,003$

$CF_3^e = +38,911$

$CF_4^e = -66,600$

The new NPV is $3,827.44, only slightly greater than the value of $3,771.84 with zero inflation. Other inflation rates will produce almost the same net present value.

Conclusion

We demonstrated in this article that traditional duration management using either floating- or fixed-rate debt leaves a company's equity value exposed to changes in interest rates. This approach ignores the important distinction between the effects of real and inflationary changes in the nominal interest rate on real asset values where cash flows are tied to inflation. Specifically, we showed that floating-rate debt leaves companies exposed to increases in real interest rates, while fixed-rate debt results in residual exposure to a decline in inflation during the life of the debt.

Companies can manage both sources of risk simultaneously by using CIPS. The popularity of treasury inflation-protected securities reflects a growing demand by investors for inflation-indexed bonds. Our results show that corporate issuers can use inflation-indexed debt to address their interest rate exposure in a way that is superior to traditional floating or fixed rate debt, while also benefiting from a lower cost of funds due to demand for inflation protection. Thus, we provide a theoretical basis for the development of the cash and synthetic markets for corporate inflation-protected securities.

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APPENDIX A

Impact of an Increase in Inflation on Net Present Value.

To see how an increase in inflationary expectations impacts on net present value, differentiate Eq. (2) with respect to π:

$$\frac{dNPV}{d\pi} = \sum_{t=1}^{T} t(I_t + P_t)(1+r_e)^{-t}(1+\pi)^{-t-1}$$

$$= \sum_{t=1}^{T}\left[\frac{t(I_t + P_t)}{(1+r_e)^t(1+\pi)^t}\right]\frac{1}{(1+\pi)} \quad . \qquad (A1)$$

Define L to be the present value of the loan payments using the nominal cost of equity as the discount rate in the calculation:

$$L = \sum_{t=1}^{T}\frac{(I_t + P_t)}{(1+r_e)^t(1+\pi)^t} \qquad (A2)$$

The Macaulay duration of the loan payments, still using the nominal cost of equity as the discount rate, is represented by D_L and is defined as follows:

$$D_L = \sum_{t=1}^{T}\left[\frac{\frac{t(I_t + P_t)}{(1+r_e)^t(1+\pi)^t}}{L}\right] \qquad (A3)$$

Thus, utilizing Eq. (A2) and Eq. (A3) and rearranging the right-hand side of Eq. (A1) gives the following expression:

$$\frac{dNPV}{d\pi} = \frac{D_L L}{(1+\pi)} \quad . \qquad (A4)$$

APPENDIX B:
Impact of an Increase in the Real Interest Rate on NPV

The impact of an increase in the real interest rate is found by differentiating Eq. (1) with respect to r_e ,

$$\frac{dNPV}{dr_e} = -\sum_{t=1}^{T} tCF_t(1+r_e)^{-t-1}(1+\pi)^{-t} + \sum_{t=1}^{T} t(I_t+P_t)(1+r_e)^{-t-1}(1+\pi)^{-t} \quad . \tag{B1}$$

Rearranging Eq. (B1) gives

$$\frac{dNPV}{dr_e} = \left(-\sum_{t=1}^{T} tCF_t(1+r_e)^{-t}(1+\pi)^{-t}\right)\frac{1}{(1+r_e)^1} + \left(\sum_{t=1}^{T} t(I_t+P_t)(1+r_e)^{-t}(1+\pi)^{-t}\right)\frac{1}{(1+r_e)^1} \quad . \tag{B2}$$

Representing the present value of the operating cash inflows by A and the duration of the operating cash inflows by D_A, their respective values are

$$A = \sum_{t=1}^{T} \frac{CF_t}{(1+r_e)^t(1+\pi)^t} \quad , \tag{B3}$$

$$D_A = \sum_{t=1}^{T}\left[\frac{\frac{t(CF_t)}{(1+r_e)^t(1+\pi)^t}}{A}\right] . \tag{B4}$$

Then, Eq. (B2) can be written as

$$\frac{dNPV}{dr_e} = -[D_A - D_L(L/A)]A\frac{1}{(1+r_e)} \cdot \tag{B5}$$

APPENDIX C:
Eliminating the Risk of Changes in the Inflation Premium.

To formally demonstrate how a firm can immunize a capital budgeting project against nominal interest rate changes, consider a company that finances a project by selling an inflation-indexed bond having the characteristics of a TIPS. Suppose the coupon rate is x and the initial face value is FV*. Over time the face value of the bond will increase with inflation, so

$$FV_t = FV^*(1+\pi)^t \quad . \tag{C1}$$

Likewise the bond's coupon payment will change over time, with the coupon payment in year "t" given by

$$PMT_t = xFV_t = xFV^*(1+\pi)^t \; . \tag{C2}$$

Assume the project's operating cash flows are expected to go up over time with inflation, so the year t nominal operating cash flow (CF_t) is equal to the expected operating cash flow measured in today's dollars (CF^*_t) inflated for t years, $CF_t = CF_t^*\,(1+\pi)^t$. Further, suppose the project is financed with inflation-indexed bonds having a maturity of T years which are sold for an amount B. Then, the project's initial cash outflow (equity-residual approach) is CF_0 + B and the nominal equity-residual cash inflow in period t < T is the operating cash flow less the coupon interest payment, $CF_t^*(1+\pi)^t - xFV^*(1+\pi)^t$. Finally, in the terminal time period the bond matures and the equity-residual cash inflow is $CF_T^*(1+\pi)^T - xFV^*(1+\pi)^T - FV^*(1+\pi)^T$. Then, substituting these expressions into Eq. (1) the net present value of the capital budgeting project (equity residual approach) is given by

$$NPV = CF_0 + B + \left(\sum_{t=1}^{T}\frac{CF_t^*(1+\pi)^t}{(1+r_e)^t(1+\pi)^t}\right) - \left(\sum_{t=1}^{T}\frac{xFV^*(1+\pi)^t}{(1+r_e)^t(1+\pi)^t}\right) - \frac{FV^*(1+\pi)^T}{(1+r_e)^T(1+\pi)^T} \cdot \tag{C3}$$

Note that the $(1+\pi)$ terms cancel out, and inflation is eliminated from the net present value calculation:

$$NPV = CF_0 + B + \left(\sum_{t=1}^{T}\frac{CF_t^*}{(1+r_e)^t}\right) - \left(\sum_{t=1}^{T}\frac{xFV^*}{(1+r_e)^t}\right) - \frac{FV^*}{(1+r_e)^T} \quad . \tag{C4}$$

The NPV now is expressed entirely in real terms, and the project is immunized against changes in interest rates attributable to a changing inflation premium. Further, by issuing debt of appropriate duration the duration gap can be made zero, and the project will then be immunized against changes in the real interest rate, as well.